Radial-Line to Coaxial-Line Junction with a Dielectrically Sheathed Post

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***Abstract*—An analysis of a coaxial line/radial line junction is presented for the case where the centre conductor of the coaxial line extends the full width across the radial line and is completely sheathed in a lossless dielectric. The problem is analyzed by the Fourier transform method using a realistic model for the coaxial excitation. Expressions for the current distribution on the post and the admittance of the junction are deduced. Comparison of theoretical and experimental results show the theory to be very accurate.**

I. Introduction

JUNCTIONS between coaxial line and waveguides are used extensively in microwave equipment. Consequently there is much interest in the analysis and modeling of such junctions in order to assist in their design.

Radial line to coaxial line junctions, and variations thereon, have been considered for the case of a homogeneous dielectric in the radial line region [1]–[3]. These analyses have lead to accurate results for the current distribution and input admittance, as well an equivalent circuit representation [2].

In order to further this topic, the problem considered here is that of a coaxial junction with the centre probe surrounded by a dielectric sheath. Two configurations are examined, firstly the case where the radial line is perfectly matched, and secondly where the radial line is short circuited at some distance from the junction.

Results from the analysis are compared to measured values for the input admittance of the junction, which shows the theory to be very accurate.

II. Analysis

A. Matched Radial Line

Consider the junction shown in Fig. 1 in which the inner conductor of the coaxial line, of radius $r = a$, extends the full width across the radial line of height h. The outer conductor of the coaxial line, and hence aperture radius in the radial line is of radius $r = b$. In the radial line the inner conductor is surrounded by a lossless dielectric sheath of relative permittivity ε_r and radius $r = c$. In the analysis presented here it is assumed that the radial line region where $r \geq c$ is filled with a lossless air dielectric.

It is further assumed that the junction is driven from the coaxial line, and that the fields are axially symmetric. It is also assumed that all metallic surfaces are perfectly conducting, and that the radial line is perfectly matched at some distance from the junction.

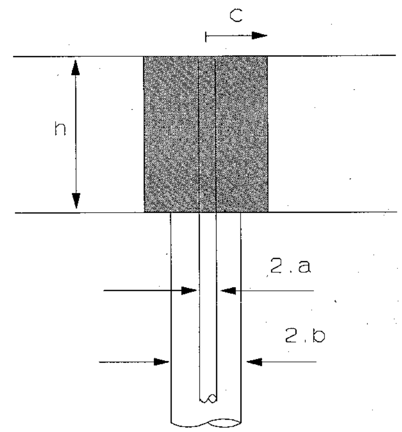

Fig. 1. Radial-line to coaxial-line junction, with dielectrically sheathed center post.

It has been shown elsewhere [2] that the fields of the junction problem are equivalent to those of an infinite cylindrical antenna with an infinite series of magnetic frills spaced at intervals of $2h$ along the antenna. The fields in one "cell" $(0 \leq z \leq h)$ of the problem are equivalent to those in the radial line. It has also been shown that the analysis of this equivalent problem is assisted by first considering a similar situation with only one frill at $z = 0$.

If the current distribution on the single frill antenna is $I^*(z)$, then the current distribution on the infinite frill antenna is easily shown to be given by

$$I(z) = \sum_{m=\infty}^{\infty} I^*(z + 2mh), \tag{1}$$

which can be shown by Poisson's Summation Formula to be

$$I(z) = \frac{1}{2h}\left\{\mathcal{I}(\alpha = 0) + 2\sum_{m=1}^{\infty}\mathcal{I}(\alpha = m\pi/h)\cos(m\pi z/h)\right\}, \tag{2}$$

where $\mathcal{I}(\alpha)$ is the Fourier transform (with respect to z) of $I^*(z)$. Fourier transform (FT) quantities are henceforth given in script, $j = \sqrt{-1}$, and the time dependence $e^{j\omega t}$ is assumed, and

$$\mathcal{I}(\alpha) = \int_{-\infty}^{+\infty} I^*(z) e^{-j\alpha z} dz. \tag{3}$$

Manuscript received October 2, 1991.
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IEEE Log Number 9106527.

The problem may be considered in terms of three regions; $a \leq r \leq b$ inside the frill region, $b \leq r \leq c$ outside the frill but inside the dielectric, and $r \geq c$ outside the dielectric. Only outward travelling fields exist in the region $r \geq c$, and the fields in the other regions can be obtained from results in [1], [2].

If the boundary conditions that $\mathcal{E}_z(r = a) = 0$, and both $\mathcal{E}_z$ and $\mathcal{H}_\theta$ are continuous across $r = c$ are applied, it is possible to show that in (4), where K_0, I_0, K_1, I_1 are modified Bessel functions (see (4) below). The constants k', η', and k, η are the wave numbers and impedances, within and outside the dielectric, and

$$q = \sqrt{\alpha^2 - k^2}, \qquad q' = \sqrt{\alpha^2 - k'^2}, \tag{5}$$

$$K_{ba'} = K_0(q'b) - K_0(q'a), \qquad I_{ba'} = I_0(q'b) - I_0(q'a). \tag{6}$$

Dashed symbols refer to quantities within the dielectric region. Equation (4) may be substituted into (2) to give the current distribution on the post.

It has been shown [1] that the admittance looking into the radial-line from the coaxial line at $z = 0$ is given by

$$Y = \frac{1}{2h}\left\{\mathcal{Y}(\alpha - 0) + 2\sum_{m=1}^{\infty}\mathcal{Y}(\alpha = m\pi/h)\right\}, \tag{7}$$

where

$$\mathcal{Y}(\alpha) = \frac{2\pi}{\eta' \ln(b/a)} \int_a^b \mathcal{H}_\theta(\alpha, 0)\, dr. \tag{8}$$

Using the results $I(\alpha)$ and the field expressions in [1], it can then be shown below in (9).

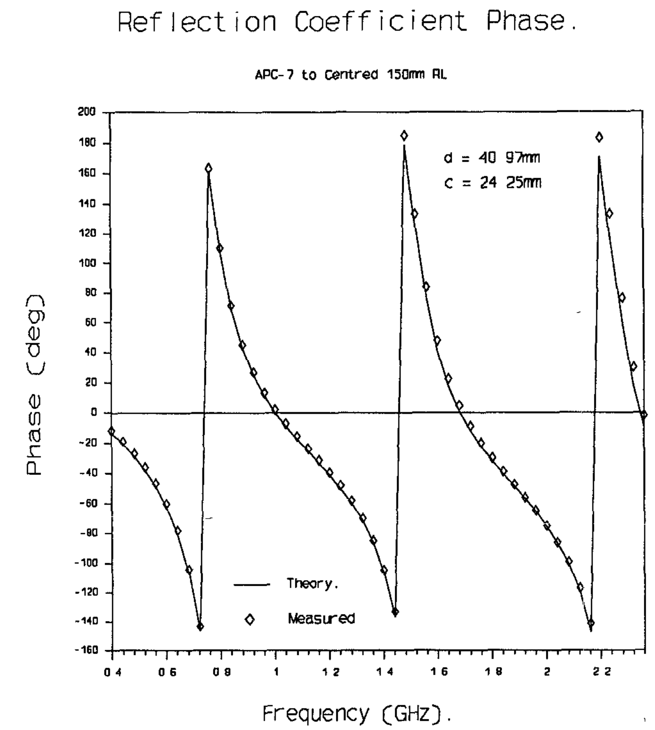

Fig. 2. Measured and theoretical results for the phase of reflection-coefficient at the input port.

B. Short-Circuited Radial Line

The analysis for the case with a short circuit located in the radial line at a radius $r = d$ (where $d \geq c$), is very similar to the matched line case. The principal difference occurs in the expressions for the fields in the region $c \leq r \leq d$, where the additional boundary condition that $\mathcal{E}z(r = d) = 0$ must be

$$I(\alpha) = \frac{-j4\pi k'}{q'^2\eta' \ln(b/a)} \cdot \frac{q'K_1(qc)(K_0(q'c)I_{ba'} - I_0(q'c)K_{ba'}) - q\varepsilon_r K_0(qc)(I_1(q'c)K_{ba'} + K_1(q'c)I_{ba'})}{q'K_1(qc)(K_0(q'c)I_0(q'a) - I_0(q'c)K_0(q'a)) - q\varepsilon_r K_0(qc)(I_1(q'c)K_0(q'a) + K_1(q'c)I_0(q'a))}. \tag{4}$$

$$\mathcal{Y}(\alpha) = \frac{j4\pi k'}{q'^2\eta' \ln^2(b/a)}\left\{\ln(b/a) + (I_0(q'a)K_0(q'b) - I_0(q'b)K_0(q'a)) \cdot \frac{q'K_1(qc)_{(K_0(q'c)I_0(q'b) - K_0(q'b)I_0(q'c))} - q\varepsilon_r K_0(qc)_{(K_1(q'c)I_0(q'b) + K_0(q'b)I_1(q'c))}}{q'K_1(qc)_{(K_0(q'c)I_0(q'a) - K_0(q'a)I_0(q'c))} - q\varepsilon_r K_0(qc)_{(K_1(q'c)I_0(q'a) + K_0(q'a)I_1(q'c))}}\right\}. \tag{9}$$

$$\mathcal{Y}(\alpha) = \frac{j4\pi k'}{q'^2\eta' \ln^2(b/a)}\left\{\ln(b/a) + (I_0(q'a)K_0(q'b) - I_0(q'b)K_0(q'a)) \cdot \frac{q'S_1(K_0(q'c)I_0(q'b) - K_0(q'b)I_0(q'c)) - q\varepsilon_r S_0(K_1(q'c)I_0(q'b) + K_0(q'b)I_1(q'c))}{q'S_1(K_0(q'c)I_0(q'a) - K_0(q'a)I_0(q'c)) - q\varepsilon_r S_0(K_1(q'c)I_0(q'a) + K_0(q'a)I_1(q'c))}\right\}. \tag{10}$$

applied. It may be shown that, for the short circuit line case, we have (10) (see preceding page), where

$$S_0 = K_0(qc) - I_0(qc)\frac{K_0(qd)}{I_0(qd)} \quad \text{and}$$
$$S_1 = K_1(qc) + I_1(qc)\frac{K_0(qd)}{I_0(qd)}. \tag{11}$$

III. Results

It is now appropriate to investigate the accuracy of the theory by comparing theoretical predictions with experimental measurements. It is very difficult to accurately match a radial-line. It is, however, straightforward to perform measurements on a short circuited radial line . It can be seen that the analysis of the short circuit case involves all the elements of the matched problem and thus, it should be a good test of the theory. For this reason the input reflection coefficient for this case was measured for comparison with the theory.

The dimensions of the test-set were $a = 1.55$ mm $b = 3.48$ mm $d = 40.77$ mm and $h = 150.0$ mm with a 7 mm 50-Ω coaxial-line input. The dielectric had an $\varepsilon_r = 2.1$ and three sheath thicknesses were examined ($c = 12.65$ mm, 16.47 mm, and 24.25 mm).

Fig. 2 shows results for the phase of the reflection coefficient (the magnitude being unity) for $c = 24.25$ mm. Clearly, there is excellent agreement between the theoretical and experimental results. The results for the other cases show similar agreement.

IV. Conclusion

The analysis of a radial-line/coaxial line junction with a dielectrically sheathed centre post has been presented and an accurate method of evaluating the current distribution on the post, and the input admittance of the junction has been formulated. The analysis began with expressions for the fields near the junction that included terms which accounted for both the coaxial excitation and reflections from the dielectric interface. Upon applying the appropriate boundary conditions, expressions for the current distribution and input admittance were found. The junction examined here is considered both in its own right, and also as a necessary first step in the analysis of related problems, such as the coaxial-line to rectangular waveguide junction.

References

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